ERASMUS PROJECT

SCHOOL YEAR 2015-16

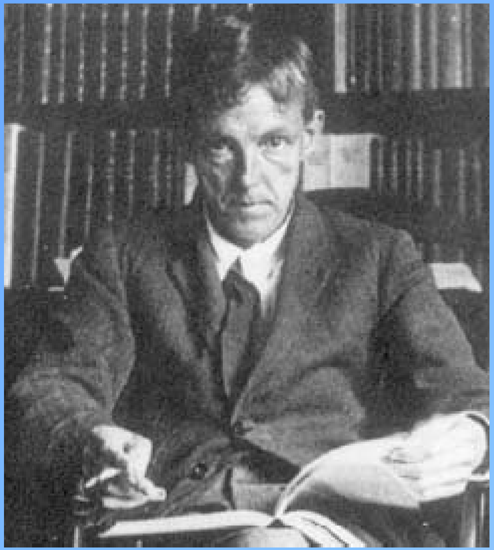
IIS LS
"PICCOLO"
CAPO D'ORLANDO

INTERDISCIPLINARY LESSON

DECORATIONS AND TESSELLATIONS

TEACHER Salvatore Tullio Randazzo (MATH AND PHYSICS)
(IIS SCIENTIFIC HIGH SCHOOL,
"L. Piccolo" Capo d'Orlando)

ENGLISH TRANSLATION Valentina Aglio (ENGLISH TEACHER)
(IIS SCIENTIFIC HIGH SCHOOL,
"L. Piccolo" Capo d'Orlando)



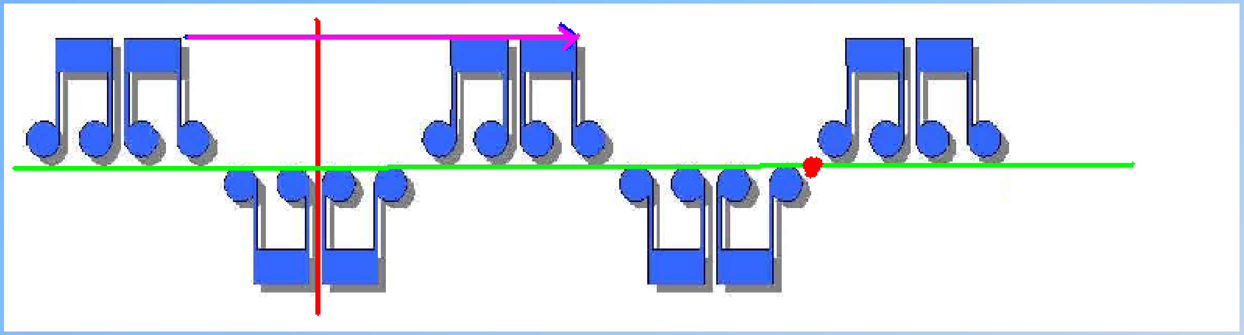
G.H. Hardy, Apology of a Mathematician, 1940

The Mathematician, as the Painter and the Poet, is a creator of shapes. If the shapes he makes last longer than the painter's and the poet's it is because his are made of ideas. The painter makes shapes with signs and colour, the poet with words…

The mathematitian, instead, has no other tools to work with but ideas. So the shapes he makes have more chances to last longer, because ideas wear themselves out less than words…

DECORATIONS

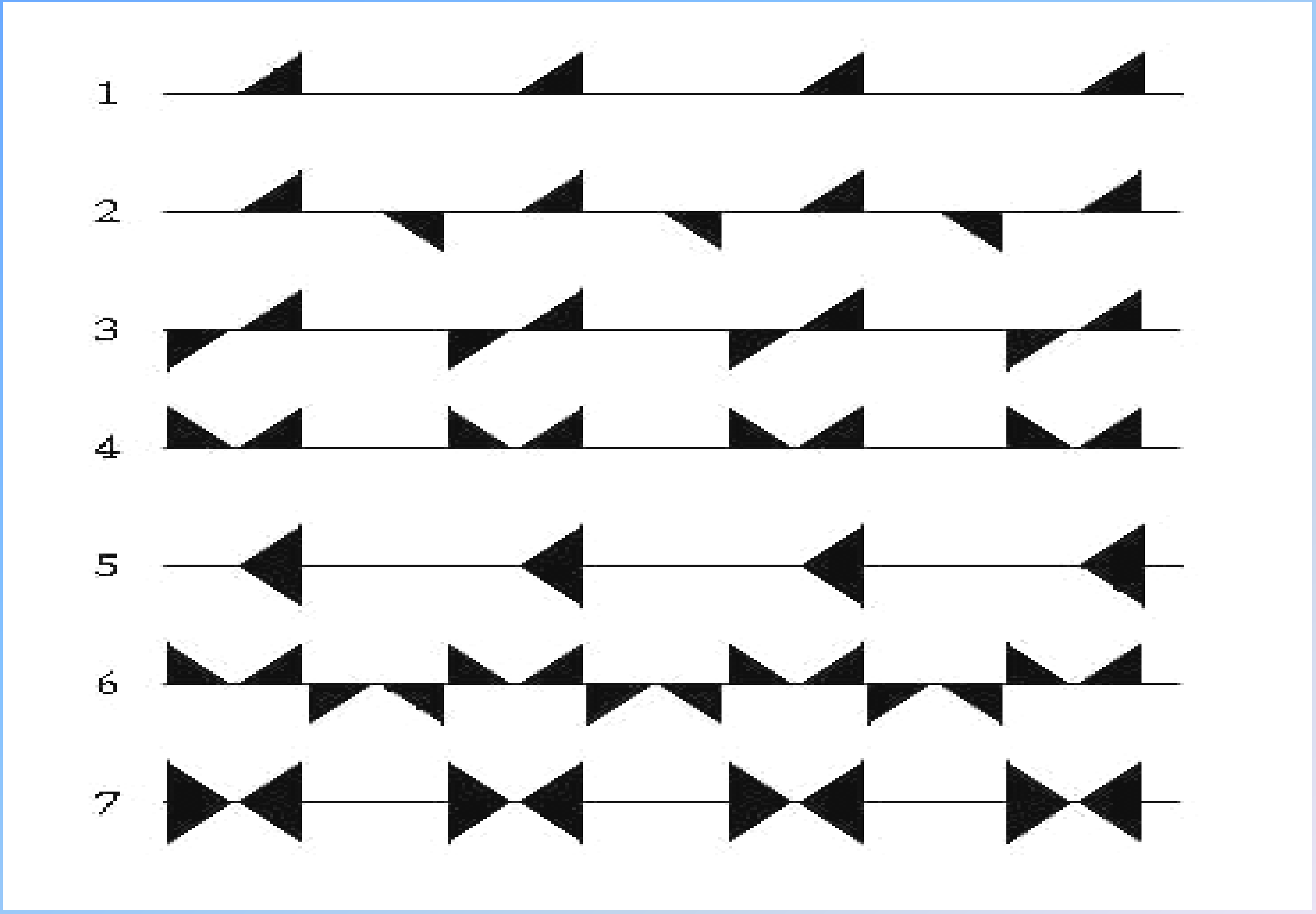
A decoration is an infinite line in which a modulus is repeated



They are a group of isometries that contain only a

- Translation;
- Rotation;
- Reflection.

We have seven types of decorations in total:



...1ST TYPE

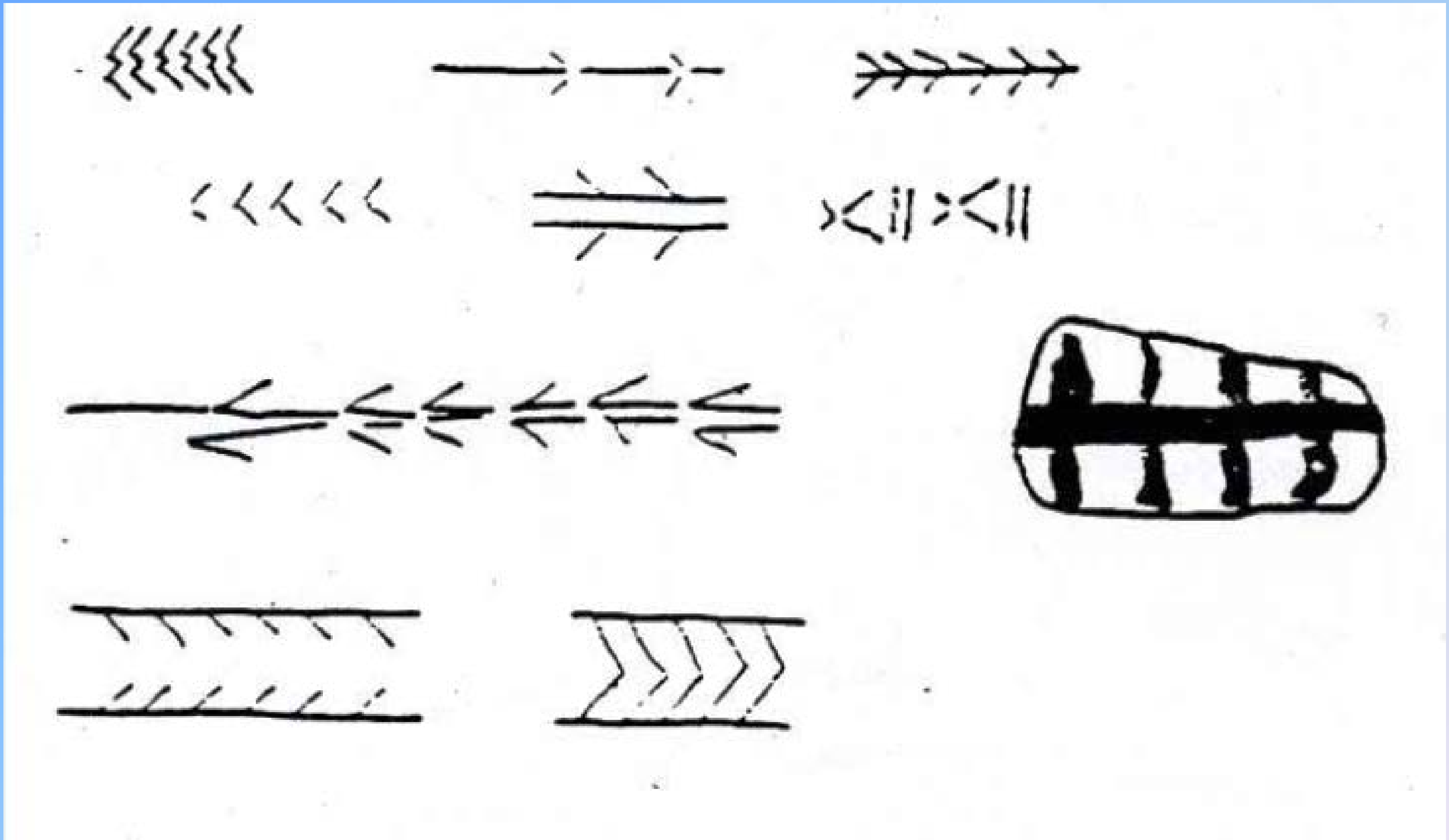
1

$$\begin{cases} x = x_0 + 1 \\ y = 0 \end{cases}$$

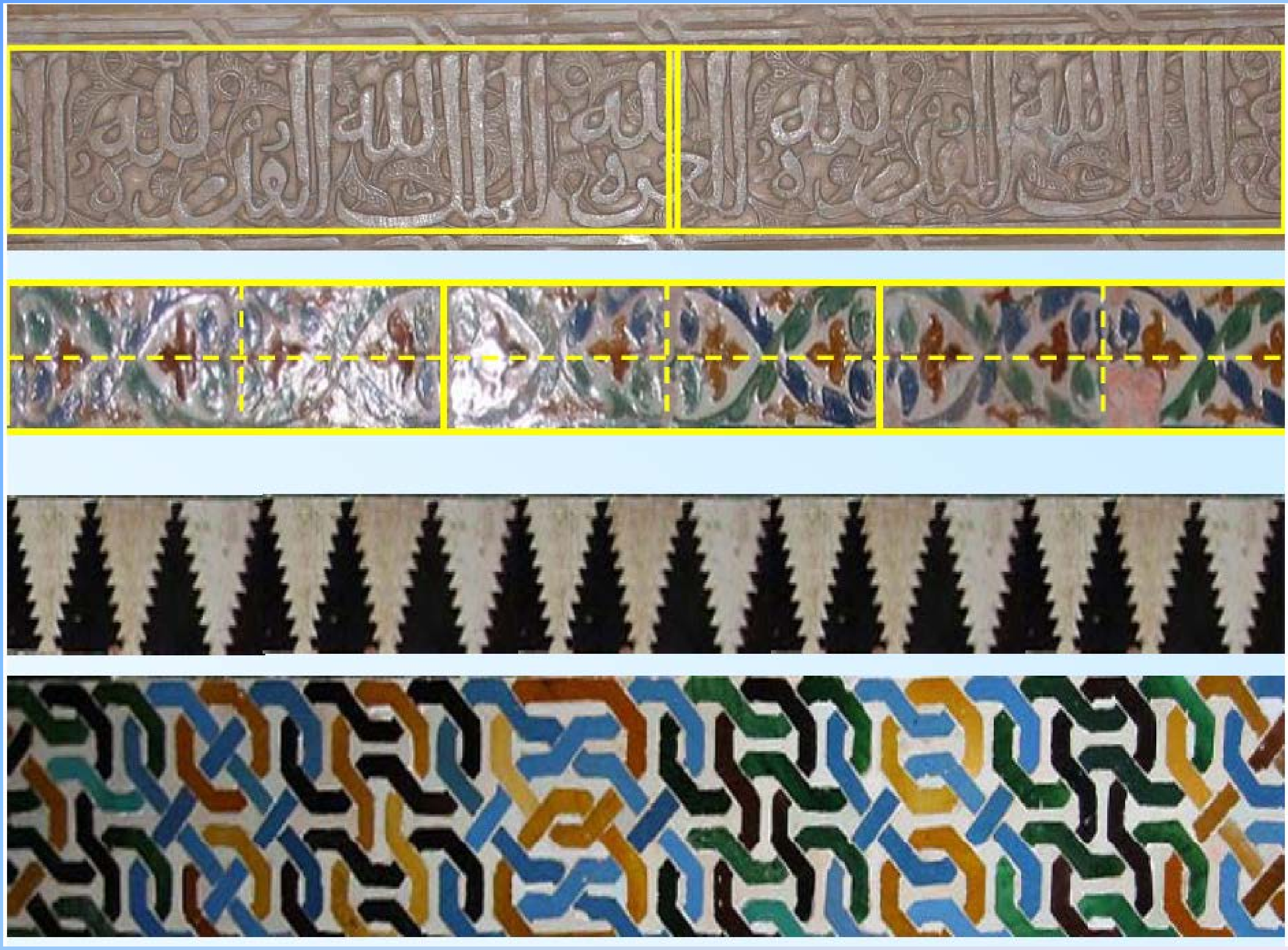
$$P_i = P_0 + i\,\vec{v}$$

$$i \in Z \qquad \vec{v}\ (1\ ,\ 0\ )$$

…and here are some examples in Art!



Paleolithic



Mosaics of Alhambra

TESSELLATIONS

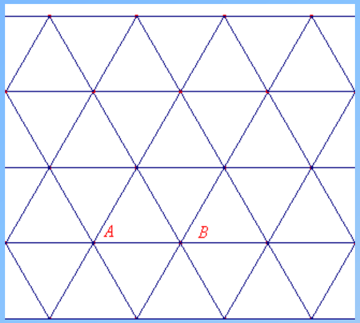
What is a Tessellation?

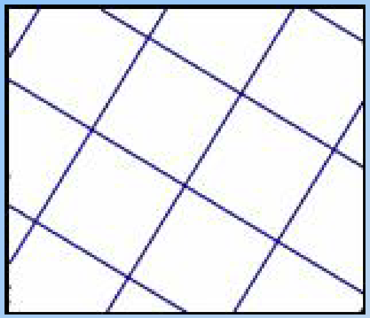
We obtain a tessellation every time a figure is periodically repeated without overlappings along two non parallel directions, so as to cover all the surface of the plane. Another way to define the tessellation is «Tiling»

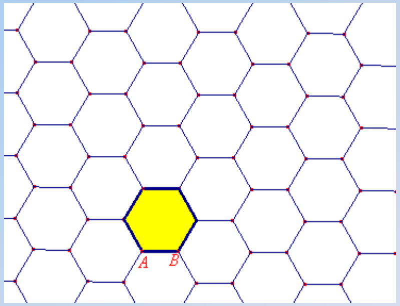
CLASSIFICATION OF TESSELLATIONS

Regular Tessellations

We can define «regular tessellation» as any tiling of the plane obtained with regular polygons which, two by two, have a side in common.



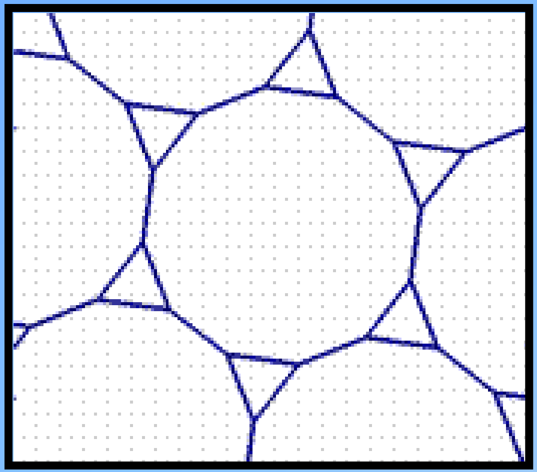




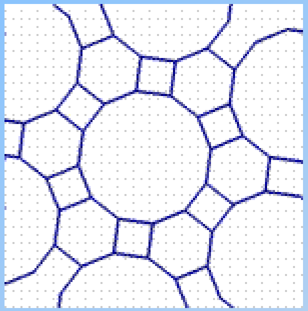
There are three types of regular tessellations: the ones built with equilateral triangles, squares and regular hexagons. That is because only in these three cases the degree measurement of the internal angle of the polygon is a submultiple of 360°. For instance, the regular pentagon does not tile the plane completely since it has got angles of 108°, that is not a submultiple of 360°.

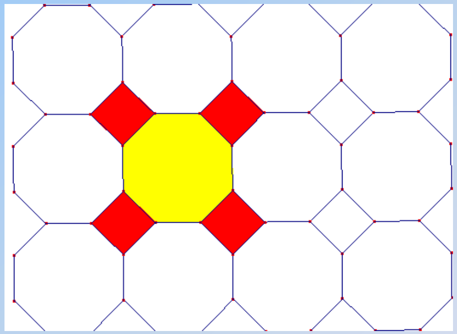
Semi regula Tessellations

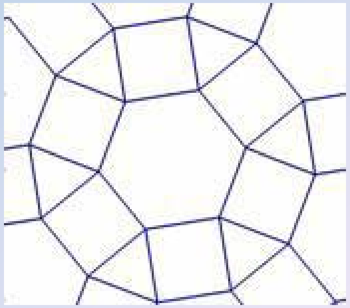
"Semi regular tessellations" or "Archimedes tessellations" are eight and they completely cover the plane with more polygons.



The tessellation shown on the side is called (3,6,3,6): which means that each vertex is alternately sorrounded by equilateral triangles and regular hexagons.







From a mathematical point of view, the tessellations of the plane can be referred to the symmetries of plane or group* shapes of plane isometries.

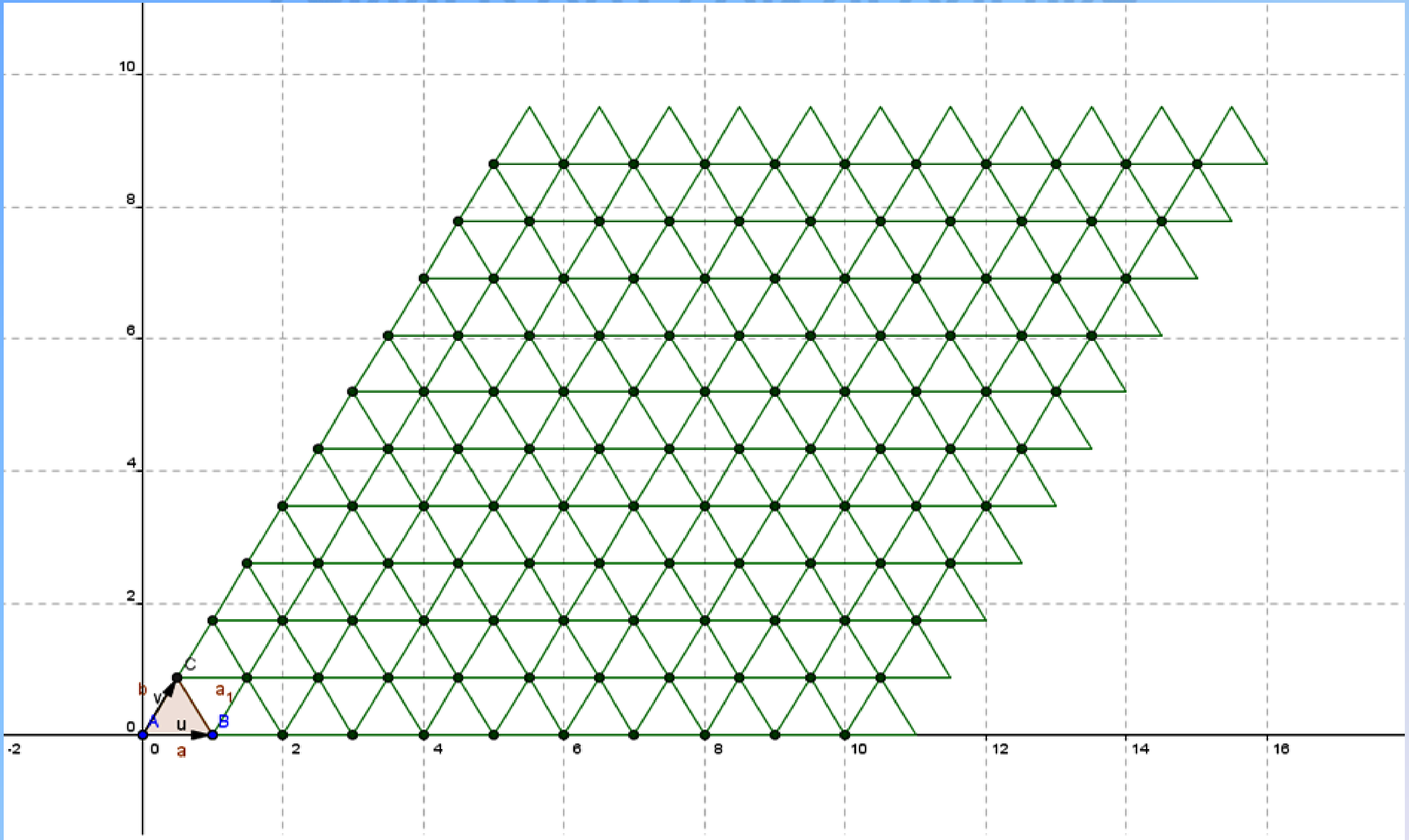
The symmetries of a geometrical object always form a group. For instance, the symmetries of a regular polygon form a finite group called **dihedral group** of 2*n* order, in which the regular polygons remain unaltered at *n* sides.

The groups associated to the sets of the tesselations' symmetries are 17.

***GROUP:** algebraic structure formed by a set with a binary operation that satisfies the following axioms: the associative property, the existence of the neutral element and of the opposite.

EXAMPLE OF A TESSELLATIONS

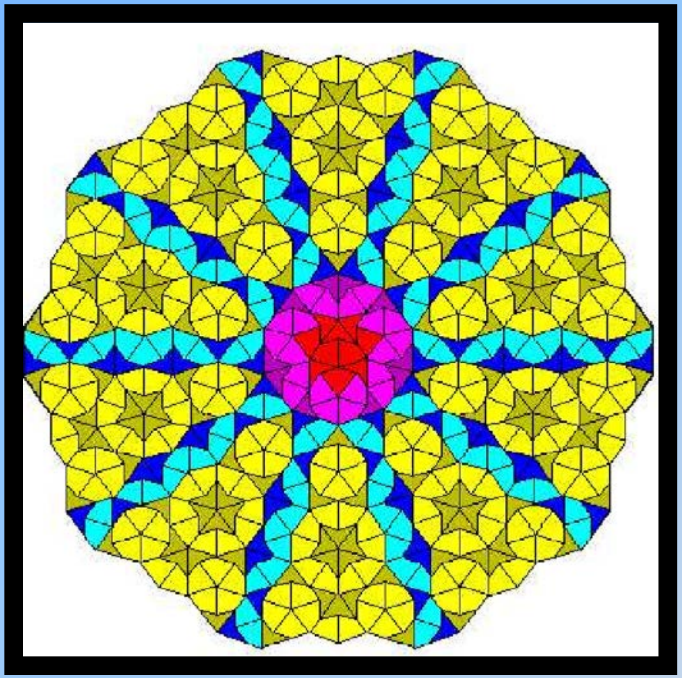
CARRIED OUT CON GEOGEBRA



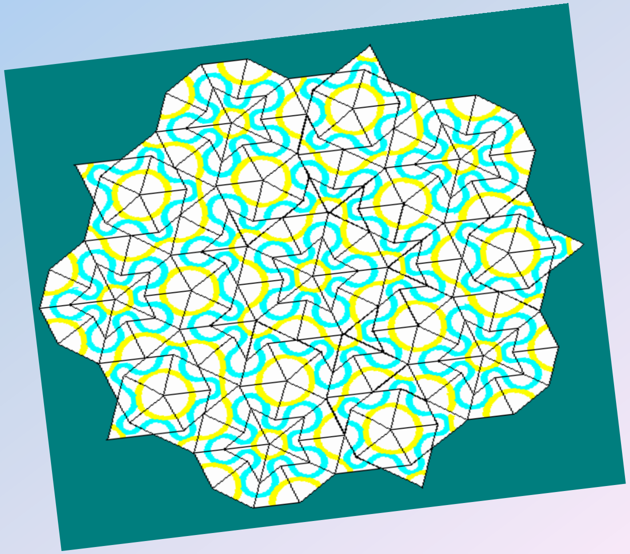
Non regular Tessellations

They are composed by infinite length bands one next to the other, each one covered by the same regular tessellation, but misaligned among them. We define as non regular tessellations also the «non recurring» ones (that is to say, without symmetry)

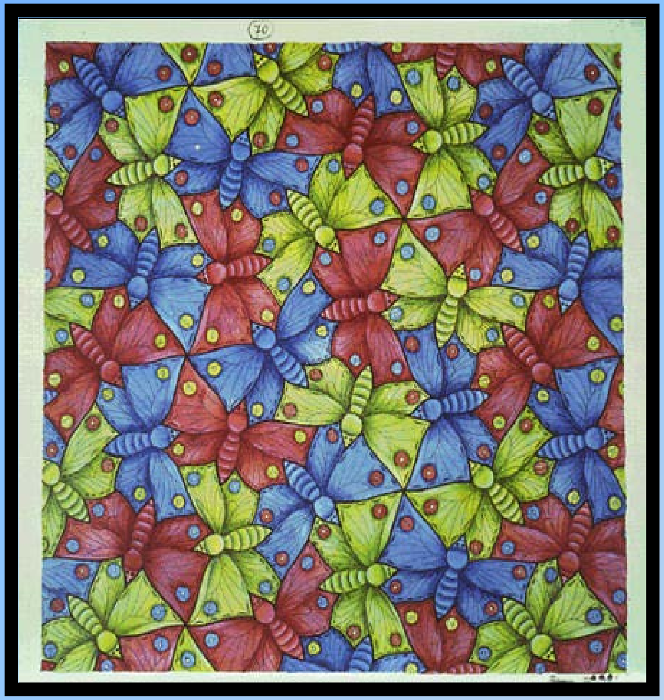
PENROSE TESSELLATION



Penrose tessellations do not contain symmetries inside them.



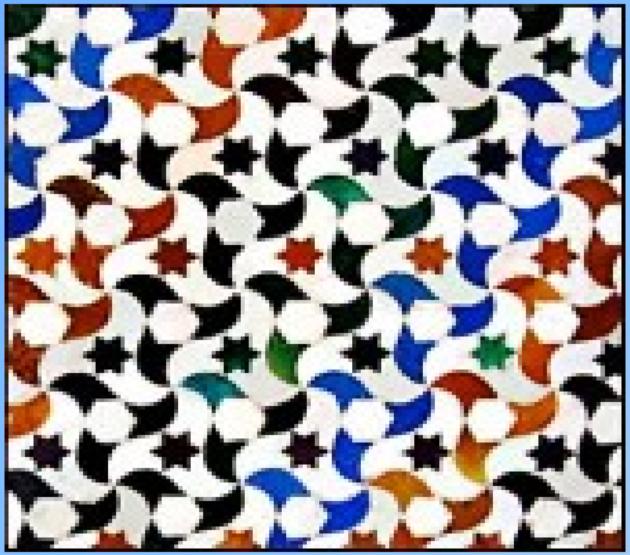
ESCHER TESSELLATIONS



Escher was fashinated by every kind of tessellation, both regular and irregular, by experimenting them even simultaneously in the works defined as metamorphosis. In them he applied reflections, translations and rotations to a great variety of shapes which, by interplaying, freed themselves, thus abandoning the position plane. Moreover, he realiszed regular shapes by screwing them and thus obtaining animals, birds and other shapes.

..and here are some examples of tessellations in art!

Tessellations in figurative, abstract art and in architecture have been a way of linking aesthetics, elegance and simplicity ever since. Some examples are:



Tesselations in Alhambra

Tessellations in Pisa Cathedral

THANKS
FOR YOUR KIND ATTENTION